USN







Second Semester B.E. Degree Examination, Jan./Feb. 2021 **Engineering Mathematics - II**

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

1 a. Solve
$$\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} - 6y = e^{3x}$$
 (06 Marks)

b. Solve
$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 3Sinx$$
 (07 Marks)

c. Solve by the method of undetermined coefficients

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 4y = 2x^2 + 3e^{-x}$$
 (07 Marks)

2 a. Solve
$$(D^4 + 4D^3 - 5D^2 - 36D - 36)y = 0$$
 (06 Marks)

b. Solve
$$\frac{d^2y}{dx^2} + \frac{dy}{dx} = x^2 + 2x + 4$$
 (07 Marks)

c. Solve by variation of parameters method
$$\frac{d^2y}{dx^2} + a^2y = \tan ax$$
 (07 Marks)

3 a. Solve
$$x^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + y = \log x$$
 (06 Marks)

b. Solve
$$xy\left(\frac{dy}{dx}\right)^2 - (x^2 + y^2)\frac{dy}{dx} + xy = 0$$
 (07 Marks)

c. Find the general and singular solution of
$$y = px - \sin^{-1}p$$
. (07 Marks)

4 a. Solve
$$(1+x)^2 \frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = \sin(2\log(1+x))$$
 (06 Marks)

b. Solve
$$p^2 + 2py \cot x = y^2$$
, where $p = \frac{dy}{dx}$ (07 Marks)

c. Solve
$$(px - y)(py + x) = a^2p$$
 by taking $x^2 = X$ and $y^2 = Y$. (07 Marks)

5 a. Form the partial differential equation from
$$xyz = \phi(x + y + z)$$
 (06 Marks)

b. Solve
$$\frac{\partial^3 z}{\partial x^2 \partial y} + 18xy^2 + \sin(2x - y) = 0$$
 by direct integration. (07 Marks)

Find all possible solutions of the one-dimensional heat equation $U_t = c^2 U_{xx}$ by the method of separation of variables. (07 Marks)



(06 Marks)

- 6 a. Form the partial differential equation from z = f(x + at) + g(x at), where a is a constant. (06 Marks)
 - b. Solve $\frac{\partial^2 z}{\partial x^2} = a^2 z$, given that at x = 0, $\frac{\partial z}{\partial x} = a \sin y$ and z = 0. (07 Marks)
 - c. With suitable assumptions, derive the one dimensional wave equation as $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ (07 Marks)

Module-4

- 7 a. Evaluate $\int_{0}^{1} \int_{0}^{\sqrt{1-y^2}} x^3 y dx dy$ by changing the order of integration. (06 Marks)
 - b. Evaluate $\int_{-1}^{1} \int_{0}^{z} \int_{x-z}^{x+z} (x+y+z) dx dy dz$ (07 Marks)
 - c. Show that $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$ (07 Marks)

OR

- 8 a. Evaluate $\int_{0}^{a} \int_{y}^{a} \frac{x}{x^2 + y^2} dxdy$ by changing to polar coordinates. (06 Marks)
 - b. Using double integration find the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$.

 (07 Marks)
 - c. Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$ (07 Marks)

Module-5

- 9 a. Find Laplace transform of t(Sinat + Cosat) (06 Marks)
 - b. Find the Laplace transform of the periodic function of period 2a given by

$$f(t) = \begin{cases} t, & 0 < t < a \\ 2a - t, & a < t < 2a \end{cases}$$
 (07 Marks)

c. Using convolution theorem find $L^{-1} \left[\frac{s}{(s^2 + a^2)^2} \right]$ (07 Marks)

OR

- 10 a. Express $f(t) = \begin{cases} Cost, & 0 < t < \pi \\ Cos2t, & \pi < t < 2\pi \end{cases}$ in terms of unit-step function and hence find L(f(t)). $Cos3t, & t > 2\pi \end{cases}$
 - b. Find the inverse Laplace transform of

i)
$$\frac{s^2 - 3s + 4}{s^3}$$
 and ii) $\frac{s + 2}{s^2 - 4s + 13}$ (07 Marks)

c. Solve by Laplace transform method $\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + x = e^t$ with x = 2, $\frac{dx}{dt} = -1$ at t = 0.

(07 Marks)